Goran Kostić

METROLOGY COMPANION

THE NEW SI E-BOOK ON CONCEPTS AND COMPUTATIONS IN MEASUREMENTS

SYMMETRY

Goran Kostić

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Metrology Companion

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Leskovac, 2019

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Preface

From a shop floor to the fundamental sciences, statistical analysis of results of repeated measurements is required. The purpose of the analysis is to determine the value of a measured quantity, as well as the accuracy of the determined value. It is requested that accuracy is described by the standard measurement uncertainty to enable traceability and ensure computing of the confidence level and the confidence interval.

The essence of measurement is the same in all areas. The purpose of this publication is to be a companion to people dealing with measurements, that is experiments or observations, from production to research, from physics and chemistry to biology and medicine. The publication is also intended for students.

In front of you is the Companion with concisely described subjects that are indispensable in all measurements. The Companion consists of about minimal and sufficient set of concepts and methods required to compute a measurement result and its measurement uncertainty. The text is in the form that enables direct applications in computations, manual or spreadsheet, and in writing software.

The manner of presentation is encyclopedic. However, the order of presentation allows reading the text as a book or textbook. It seems that the form of encyclopedias is suitable for expert writings because it facilitates understanding of relations among concepts that make the essence of every exact knowledge.

Most of this text is a terminologically consistent and compliant synthesis of international recommendations and regulations, articles and encyclopedic texts. References are given in brackets.

Special attention was paid to the definitions of terms and their consistent use. Most of the terms are according to the second and third editions of the International vocabulary of metrology (see [VIM 2] and [VIM 3]) which was issued by the Joint Committee for Guides in Metrology (JCGM). These glossaries were not applied when it was thought that they do not correspond to what they should express, or that there are far better terms. Any such exceptions from the glossaries are noted with the references.

The second part of the Companion, regarding statistical analysis of measurement results, is the basis for computation of the standard measurement uncertainty. It is intended for anyone involved in the analysis of the measurement results.

The chapter on the standard measurement uncertainty is according to the instructions published by the JCGM in the Guide to the expression of uncertainty in measurement (see [GUM]).

The level and interval of confidence of the measurement result can be considered as a purpose of determination of all measurement errors. An accurate evaluation of the level and interval of confidence is enabled by the standard measurement uncertainty of a traceable value. In this Companion, the traceability is described in the short, but comprehensive text given at the end of the section concerning measurement uncertainty.

At the end of the Preface, attention is drawn to the following viewpoint expressed in [GUM]. “The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the measurement result therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value.”

All equations in the Companion are for the coherent units.

The following marking is used in the Companion.

- Bold fonts are used for terms when defined.
- The abbreviated terms that may be used are provided next to the terms, in parentheses, when this will not lead to confusion.
- Underlined text is linked to detailed explanations of a concept which underlined text indicates. A “see...” is also linked to a reference.
- Equations, tables and figures are marked with the number of the chapter, then by a point, and subsequently by their order number in the chapter.
- A blue text needs special attention.

For the help in the creation of the Companion, I wish to thank to engineer *Siniša Hristov*, professor Dr *Gligorije Perović*, Dr *Walter Bich*, professor Dr *Dragić Banković*, professor Dr *Vesna Jevremović* and Dr *Emina Krčmar*. To my longtime colleague, engineer *Zlatko Sudar*, I am grateful for comments which have led to an improved text.

I am grateful to professor *Ljiljana Sudar* and professor Dr *Živomir Petronijević* for their efforts and valuable remarks of the trial readers.

Thanks to *Ljiljana D. Kostić* for her trust and material support.

This Companion was created as a part of the development project of measurement and control equipment, which is in progress in the laboratory for development of electronic products, Symmetry, based in Leskovac, Serbia.

This is an updated e-edition that comes after the fundamental revision of the International System of Units (SI) as well as one English and two Serbian editions.

Please send comments to the symmetry@ptt.rs.

Leskovac, 15th August 2019

Goran Kostić

Introduction

The measurements are essential for human activity - from the trade to the fundamental sciences.

International trade has been one of the important reasons for the establishment of the International Bureau of Weights and Measures (BIPM). The Bureau contributes to the reduction of technical barriers to trade, ensuring that measurements and tests carried out in different countries are considered equivalent.

Scientists create their theories on a basis of measurement results which are for them facts about the material world, true within limits of measurement errors.

The International System of Units was conceived after ancient discussions of European scientists about the need for a new system of measurement. The first proposal of a unified system of units, based on the metre (metric) and decimal, was made as early as 1670 by *Gabriel Mouton*, a priest from Lyon, France. That system replaced national and regional variants that made scientific and commercial communication difficult. The first decimal metric system was established in France in 1799 as an important fruit of the French Revolution. The world became metric in early 1993 when the International System of Units (SI) became primary in the USA government institutions.

The current SI has emerged as perhaps the most significant revision of the SI. This revision was voted at the 26th General Conference on Weights and Measures (CGPM) in November 2018. The current definitions of SI base units are fundamentally different from those previously used. Now, the seven SI base units are defined using only seven “defining constants” whose values are taken as exact. The definitions are published in the 9th edition of the SI Brochure and are applied from 20th May 2019.

The International Bureau of Weights and Measures was created by the Metre Convention signed in Paris on 20th May 1875. The Convention was amended in 1921 and has remained the basis of the international agreement on measuring units until now. The original signatories of the Convention were representatives of 17 States, and on 15th August 2019, there were 60 Member States of the Convention and 42 Associates of the

CGPM. Since its establishment, the headquarters of Bureau is in Sevres near Paris.

The purpose of Bureau is to ensure worldwide unification of measurements in order to enable traceability of measurement results to the International System of Units. Therefore the task of Bureau is to:

● "represent the worldwide measurement community, aiming to maximize its uptake and impact

● be a centre for scientific and technical collaboration between the Member States, providing capabilities for international measurement comparisons

● be the coordinator of the worldwide measurement system, ensuring it gives comparable and internationally accepted measurement results."

The Bureau operates under the exclusive supervision of the International Committee for Weights and Measures (CIPM) which itself comes under the authority of the General Conference on Weights and Measures. The International Committee and the General Conference was established by the Metre Convention, at the same time as the Bureau.

By harmonizing and providing complete coverage of all areas, the International System of Units is an outstanding tool of the modern sciences and practices that created it.

[SI] Appendix 4. Part 2, 1.1, PP 117 [Britannica] [NIST]

1 Quantities and units of measurement

1.1 Measurable quantity

Measurable quantity (**quantity**) is a **property** of a body, a substance, or a phenomenon, that may be distinguished qualitatively and determined quantitatively.

Quantity is not a body, a substance or a phenomenon.

Quantity is independent of measurement and system of quantities.

The term quantity may refer to a quantity in a general sense (length, time, mass, temperature...) or to a value of a particular quantity (the length of a given rod, the electric resistance of a given wire, a number of entities...).

[VIM 2] 1.1 [VIM 3] 1.1 [Sonin] 2.3

1.2 Quantities of the same kind

Quantities of the same kind are quantities whose values may be added or subtracted, but that it has a physical representation.

Examples. The addition of values of all forms of energies, such as the amount of heat, kinetic energy and potential energy, has a physical representation. Thus, all forms of energies, are quantities of the same kind. A value of energy and a value of moment of force have the same dimension but there is no physical representation of addition or subtraction of their values, so those quantities are not of the same kind.

In a given system of quantities:
- quantities of the same kind have the same dimension
- quantities of the same dimension are not necessarily of the same kind
- quantities of the different dimension are always of a different kind.

Quantities of the same kind are grouped into **categories of quantities**. Examples of the categories of quantities are:
- thickness, circumference, wavelength
- heat, work, energy.

[Quinn] [VIM 2] 1.1 Notes [VIM 3] 1.2, 1.7

1.21.1 SI base units

Seven SI base units are defined using only the seven defining constants. The values of defining constants were fixed after being identified as the best estimates at the time just before the decision on the revision of the SI. In this way the SI is scaled to the seven physical constants of high reproducibility, and these constants become exact. See Figure 1.1 and Table 1.1. The reference definitions of base units are published in the SI Brochure, see [SI].

The current definitions of SI base units have brought important benefits. The most important are: 1) a large number of physical constants become either exactly known or known with higher accuracy; 2) values of quantities that are much smaller or much larger than the base units can be measured with unreduced accuracy; 3) liberation of mass metrology from the unit based on the mass standard which is undoubtedly changing with time; 4) the Josephson effect and Hall effect can be used to directly realize the SI definitions of most electrical units; 5) the definition of the kelvin does not make reference to a water having the defined isotopic composition.

Some of the consequences of the current definitions are that the following constants are not exactly known, and must be measured: 1) the magnetic constant, the electric constant and the characteristic impedance of a vacuum; 2) the temperature of triple point of water; 3) the molar mass of carbon 12.

The current definitions of SI base units are fundamentally different from those previously used. However, as it was the case in the past revisions of the SI, the transition to the current definitions was realized without noticable impact on daily life, while the measurements based on previous definitions of the units remain valid within their measurement uncertainties.

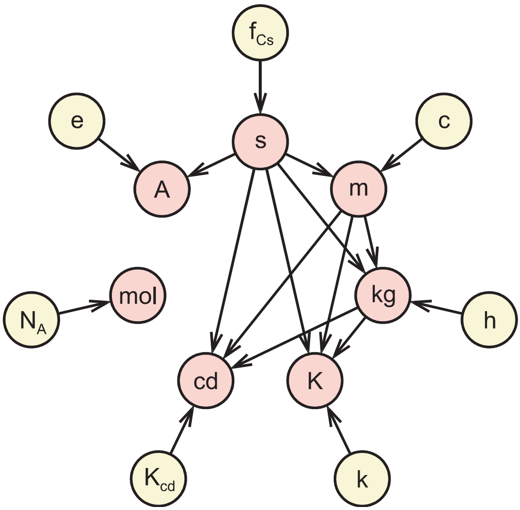

Figure 1.1. The SI base units are defined using the set of only seven defining constants whose values are taken as exact. Each base unit is defined, either, through the corresponding constant from the set, or through the constant from the set and other base units that are also defined through the constants from that set.

Table 1.2 SI coherent derived units with special names and symbols
[SI] 2.3.4

SI derived quantity	Special name of SI unit	Special symbol for SI unit	Expressed in terms of other SI units	Expressed in terms of SI base units
plane angle	radian	rad		$m \cdot m^{-1} = 1$
solid angle	steradian	sr		$m^2 \cdot m^{-2} = 1$
frequency	hertz	Hz		s^{-1}
force	newton	N		$kg \cdot m \cdot s^{-2}$
pressure, stress	pascal	Pa	N / m^2	$kg \cdot m^{-1} \cdot s^{-2}$
energy, work, amount of heat	joule	J	$N \cdot m = W \cdot s$	$kg \cdot m^2 \cdot s^{-2}$
power, radiant flux	watt	W	J / s	$kg \cdot m^2 \cdot s^{-3}$
electric charge, amount of electricity	coulomb	C	$F \cdot V$	$A \cdot s$
(electric) potential difference, electromotive force, voltage, (electric) tension	volt	V	W / A	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
electric resistance	ohm	Ω	V / A	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
electric conductance	siemens	S	A / V	$kg^{-1} \cdot m^{-2} \cdot s^3 \cdot A^2$
electric capacitance	farad	F	C / V	$kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$
electric inductance	henry	H	Wb / A	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$
magnetic flux	weber	Wb	$V \cdot s = T \cdot m^2$	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$
magnetic flux density, magnetic induction	tesla	T	$Wb / m^2 = N / (A \cdot m)$	$kg \cdot s^{-2} \cdot A^{-1}$
Celsius temperature	degree Celsius	°C [4)]	K [4)]	K [4)]
luminous flux	lumen	lm	$cd \cdot sr$	$m^2 \cdot m^{-2} \cdot cd = cd$
illuminance	lux	lx	lm / m^2	$m^{-2} \cdot cd$
activity referred to a radionuclide	becquerel	Bq		s^{-1}
(ionizing radiation) absorbed dose, specific (imparted) energy, kerma	gray	Gy	J / kg	$m^2 \cdot s^{-2}$
(radiation) dose equivalent	sievert	Sv	J / kg	$m^2 \cdot s^{-2}$
catalytic activity	katal	kat		$mol \cdot s^{-1}$

[4)] The units °C and K are equal only when expressing temperature difference or width of temperature interval. The numerical value of a Celsius temperature expressed in degrees Celsius is related to the numerical value of the thermodynamic temperature expressed in kelvins by the relation: $t\,[°C] = T\,[K] - 273.15$. [SI] 2.3.1 The kelvin

2 Measurements

2.1 Measurement

Measurement is a determination of a value of a quantity by its comparison with the known value of the quantity.

The value of a quantity may be measured with very high accuracy except perfect.

The following are necessary for measurement: a specification of measurand, a description of measurement procedure and a device for measurement.

Measurement does not apply to nominal properties. Examples: sex of a human being; ISO two-letter country code; the sequence of amino acids in a polypeptide...

[Sonin] 2.3 [GUM] 3.1.1 [VIM 2] 2.1 [VIM 3] 2.1, 1.30 Examples

2.2 Metrology

Metrology is the science of measurements.

Metrology includes all theoretical and practical aspects of measurements, whatever the measurement accuracy and field of application.

Legal metrology is the part of metrology relating to measurements specified in the law.

The scope of legal metrology commonly includes:

● protection of the interests of individuals and enterprises

● protection of national interests

● protection of public health and safety, especially those related to the environment and medical services

● meeting the requirements for commerce and trade.

[VIM 2] 2.2 [VIM 3] 2.2 [VIML] 1.03

3 Devices for measurement

3.1 Device for measurement

Device for measurement is a device, or combination of devices, designed for measurement.

The device for measurement may be a measuring instrument or a measuring transducer. A **measuring instrument** is an indicating measuring instrument or a material measure. A material measure is: a reference material, weight, volume measure, gauge block, standard electric resistor, standard signal generator...

Devices for measurement arranged in approximate order of increasing complexity are: a part, material measure, reference material, measuring transducer, measuring instrument, measuring chain, measuring system, measuring installation.

Device for measurement is periodically checked, or qualified, by a user or a recognized laboratory. The checks determine whether an error of a device for measurement, in operating conditions, is within a specified interval and whether it will be within this interval in future.

[VIM 2] 4 introduction, 4.1, 4.3, 4.6, 4.2 [VIM 3] 3, 3.1, 3.7, 3.3, 3.6

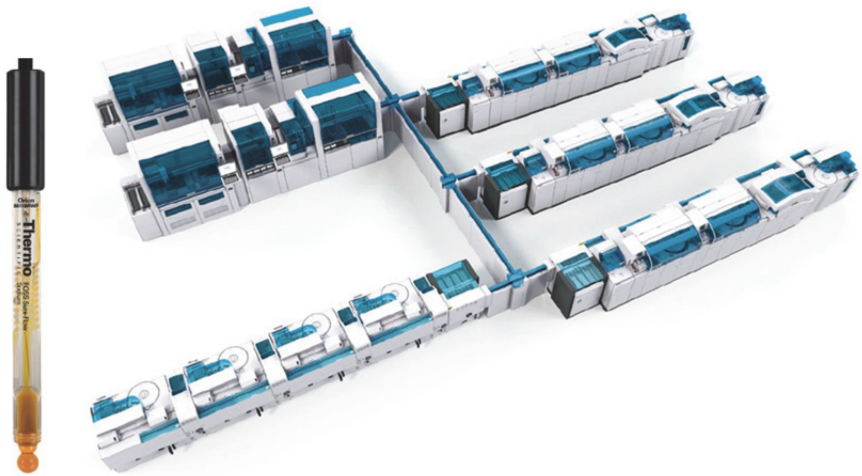

Figure 3.1. A measuring transducer and a measuring installation. Left: an ion-selective probe for concentration of sodium measurement (Thermo Scientific). Right: a highly automated measuring installation, 18 m long, for biochemical analysis in a hospital laboratory (Roche Diagnostics).

4 Measurement results, accuracy and errors

4.1 Measurement result

Measurement result (or **result of measurement**, or **estimated value**) is a value assigned to a measurand on the basis of one or more results of measurements.

A measurement result is obtained in the manner specified in the description of the measurement procedure: ● taking a **result of a single measurement** (or **particular result**), ● computing the arithmetic mean of the results of repeated measurements, ● taking the value that appears most often in the results of repeated measurements (that is, taking the mode), ● computing on the basis of a functional relationship between the value of a measurand and **component values** (or **component results**, or **component variables**, or **component quantities**, or **input quantities**, or **components**) on which the value of the measurand depends...

Results of repeated measurements of the same quantity are represented by the random variable which is characterized by statistical parameters.

A complete measurement result includes information about its accuracy.

[VIM 2] 3.1 [GUM] [VIM 3] 2.9

4.2 Accuracy

Accuracy of a value is a qualitative concept which indicates a closeness of this value to a reference value.

Accuracy of a measurement result (**accuracy**) is a qualitative concept which indicates a closeness of this measurement result to a value of the measurand.

Accuracy of a measurement result is determined in the manner specified in the description of the measurement procedure.

5 Measurement standards and calibrations

5.1 Measurement standard

Measurement standard (**standard**, or **etalon**) is a device for measurement intended to realize, reproduce or define the value of a quantity to serve as a reference in measurements.

Examples: a Kibble balance (or a watt balance); 1 kg mass standard; 100 Ω standard resistor; standard ammeter; caesium frequency standard; hydrogen reference electrode; reference solutions of cortisol, in human serum, with specified concentration.

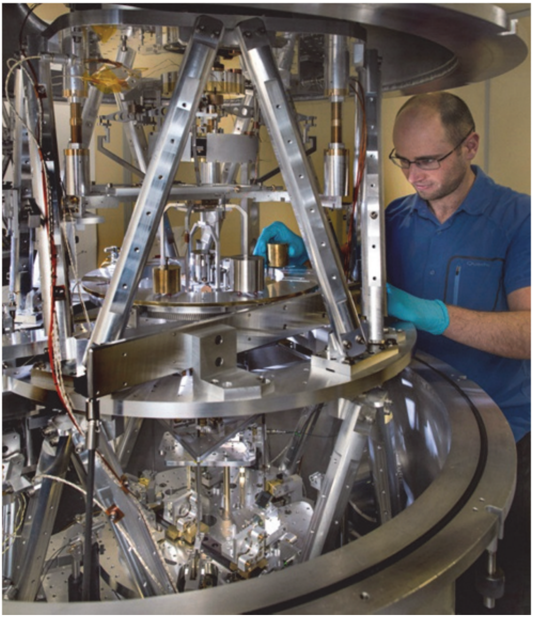

Figure 5.1. Left: a Kibble balance in the International Bureau of Weights and Measures. Among other things, the balance is used for measuring a 1 kg mass standard with the combined standard measurement uncertainty of $2 \cdot 10^{-8}$. The measurements are performed through the Planck constant, gravitational acceleration, velocity, plus electromagnetic quantities, and represent the realization of the definition of the kilogram. Right: a Josephson voltage standard used for driving a stable current through the coil of the above-mentioned Kibble balance using a standard resistor. (Courtesy of the BIPM.) [BIPM]

6 Outlines of the statistics

6.1 About statistical analysis

Statistical methods are used to determine the representative value of an investigated property for a group of **elements**. The elements within the group can have significantly different values of the property.

Evaluation (or **estimation**, or “**statistic**”) is a value **estimated** by the statistical method.

Population is a group of all elements for which we determine the representative value of a property by the statistical method. The population has a finite or infinite number of elements.

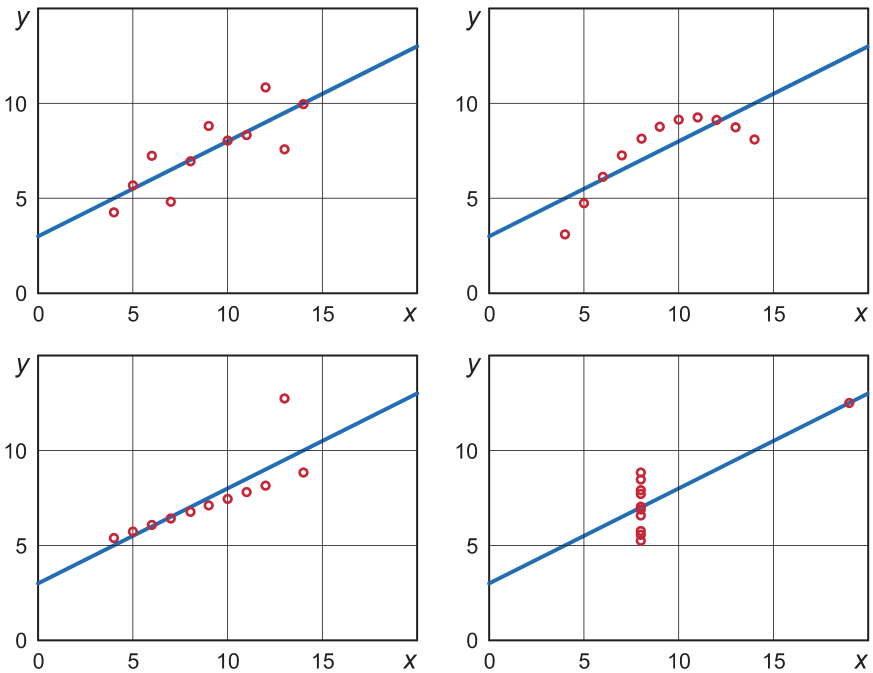

Figure 6.1. The example of four samples of values which yield: the equal arithmetic means of variables *x* and *y*, the equal deviations of *x*, the same linear regression function obtained by the method of least squares... [Anscombe]

The investigated property of a population is determined by the statistical method usually on a basis of a sample of the population that truly represents the population. The sample truly represents the population when a number of its elements is sufficiently large and when the elements are randomly selected from the population.

In order to prove that the conclusions drawn from the sample are with a certain probability valid for the population can be proved by goodness-of-fit tests.

In order to reduce the possibility of errors in statistical analysis, data graph should be necessarily considered. The graph provides an overview of qualitative features of data. The diagrams in Figure 6.1 demonstrate the importance of graphical presentation and visual analysis of data.

A property of a population determined by a statistical method is the property of a large number of elements of that population. On a basis of statistical evaluation, we can determine only the probability that a particular element of a population has a certain property.

Statistics is subjective; statisticians try to explain or predict the material world through arbitrary, but reasonable way, using probability theory, mathematics and common sense.

Unlike the statistics, probability theory for a fully defined problem gives a unique and a reproducible solution. **Probability theory** is the algebra of inductive reasoning. (Boolean algebra is the algebra of deductive reasoning.)

Statistical methods are among the most widespread quantitative methods, with important applications in almost all human activities.

[Njegić] 1.1., 1.2. [Ivković] II.1 [Britannica] Probability theory [Drake] 7-1, 7-6 [Lazić] I, 1.2 [Anscombe]

6.38 Histogram

The sample of values can be splitted into disjoint subintervals of equal widths (or **cells**, or **bins**). The **histogram** is the graphic representation of a number of the values that fall into each of the subintervals.

The histogram is made in the following way. First, split a part of the horizontal axis which contains the values from the sample into disjoint subintervals of equal width, see Figure 6.14 b). Then draw rectangles whose bases overlap subintervals on the horizontal axis and whose heights are proportional to the number of values that fall into the subinterval of their base.

The histogram appearance is significantly affected by the subintervals width and their position along the axis. The limits of subintervals can be determined according to the following two recommendations.

a) If we make the histogram for a sample with a small number of possible discrete values, the subintervals should be chosen so that each discrete value corresponds to one subinterval. This is the case in Example 6.38.1, in which 8033 discrete measurement results have one of 16 possible discrete values, out of which only 5 values are distinguished by the number of appearances. Histograms, such as in this case, are bases for reliable estimating.

b) If we make the histogram for a sample with a large number of possible values, subintervals width and position in the horizontal axis can be determined as follows.

If the n values in the sample have the normal distribution and n is larger than about 25, the optimal subinterval width w, is given by formula (6.116) with the parameter C = 3.5. The standard deviation of values in the sample is denoted by s. This formula is derived from the criterion of the minimum of integrated mean squared error (IMSE).

$$w = C \cdot \frac{s}{\sqrt[3]{n}} \tag{6.116}$$

If the values in the sample do not have the normal distribution, and n is larger than about 25, based on the experience we can also recommend using (6.116) with C = 2.

In the case of discrete values, it is necessary to round the subintervals width so that each subinterval contains the same number of possible values (that is, round subintervals width to an integer multiple of a resolution).

The limits of subintervals should be chosen so that more of the values are around the middle of subintervals.

In the histograms described in b), it is classified the case in Example 6.38.2, in which 78 discrete measurement results have one of 30 possible values. This example shows how the subintervals width affects the histogram appearance.

In most statistical software a default subinterval width and position are not optimal.

The height of a rectangle of a histogram is proportional to the probability that values fall into the subinterval of the rectangle base, so the histogram is the variation of the probability density function.

By visual inspection of the well-made histogram, we can estimate the following properties of the distribution: a type; a mode; an existence of outliers; a dispersion of values; an existence of several maximums; a symmetry; and a peakedness. The lack of an evaluation based on the histogram is subjectivity.

The approximate arithmetic mean of n values represented by a histogram, $\dot{x}$, is given by formula (6.117) derived from (6.19). The b_i is a number of values in a subinterval with the middle x_i.

$$\dot{x} \approx \frac{1}{n} \cdot (b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_n \cdot x_n) = \frac{1}{n} \cdot \sum_{i=1}^{n} b_i \cdot x_i \qquad (6.117)$$

[Britannica] Statistics [Perović, communication] [Scott] [Hristov, communication]

6.38.2 Example

Figure 6.15 a) shows the discrete values in the sample which consists of measurement results of difference of the resistances of 78 resistors. For the sample the histograms are made with different parameters, and shown in the figures b), c) and d).

The subintervals are chosen according to Section 6.38, so that they are disjoint and of equal width approximately defined by (6.116), but so that each subinterval contains the equal number of possible values. The parameters *C* and *w* are from (6.116), *n* is the size of the sample, $\overline{R}$ and R_{mode} are the means of the sample, and *s* is the standard deviation of the sample.

It can be concluded that the sample has asymmetric distribution, most probably lognormal. Also, that the histogram in the figure b), with the parameters recommended in Section 6.38 in b), has better approximation of density of values than the two other. See also 6.40 Filtering.

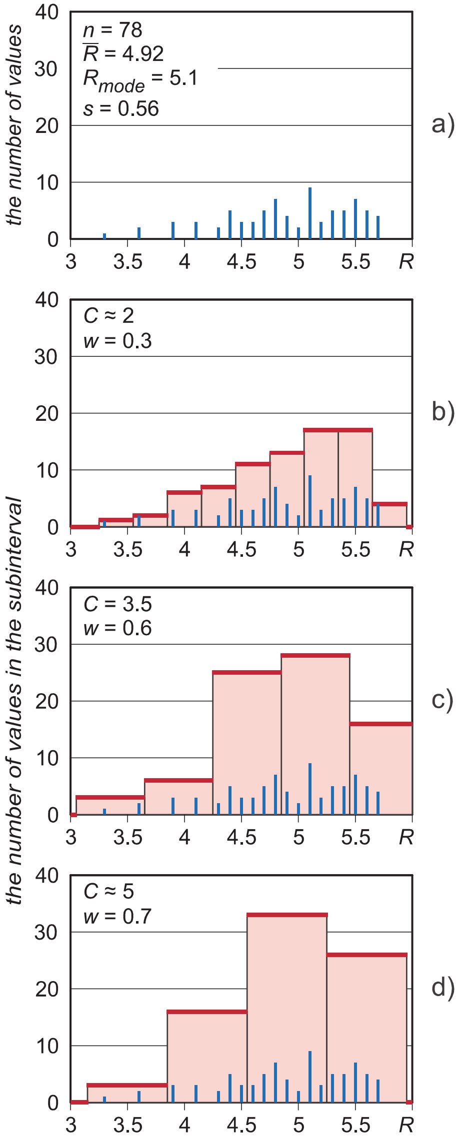

Figure 6.15. For the same sample of values, the histograms with different parameters.

7 Variances

7.1 Natural variance

Natural variance (**variance**) $\dot{V}$, of a group of m values x_i (i = 1, 2, ..., m), is an arithmetic mean of squared differences of these values from an accurate arithmetic mean of these values, $\dot{x}$:

$$\dot{V} = \frac{\sum_{i=1}^{m}(x_i - \dot{x})^2}{m}. \tag{7.1}$$

The natural variance $\dot{V}$, of a group of m values x_i, can be computed also from the squared differences of these values from pairs obtained by combinations (without repetition) of second order from m elements:

$$\dot{V} = \frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_i - x_j)^2}{m^2}. \tag{7.2}$$

Population variance (**variance of a population**, or **expected variance**) σ^2, of all m values in a population, x_i (i = 1, 2, ..., m), is computed from (7.1) by taking for the accurate arithmetic mean of these values, $\dot{x}$, the accurate arithmetic mean of a population, μ:

$$\sigma^2 = \frac{\sum_{i=1}^{m}(x_i - \mu)^2}{m}. \tag{7.3}$$

The variance $\dot{V}_m$, of values x_i which arrive continuously, is computed by the recursive method using (7.4) and (7.5), derived from (6.19) and (7.1). This computation does not require memorizing all m received values, but only the last x_m, and the previously computed arithmetic mean $\dot{x}_{m-1}$ and the variance $\dot{V}_{m-1}$.

$$\dot{x}_m = \frac{(m-1)\cdot \dot{x}_{m-1} + x_m}{m}. \tag{7.4}$$

$$\dot{V}_m = \frac{m-1}{m}\cdot \dot{V}_{m-1} + \frac{m-1}{m^2}\cdot (x_m - \dot{x}_{m-1})^2 \tag{7.5}$$

Computation using (7.5) in practice gives more accurate results than (7.1).

Using (7.5) is especially appropriate in the following cases: ● when the values for which the variance is computed arrive continuously; ● when the values have a small relative deviation; ● if it is required to determine the variance during a process (for example during Monte Carlo simulation, in which trials are stopped when the variance of the simulation results becomes small enough).

The degrees of freedom of the variance is equal to the number of values: $\nu = m$.

The variance is an indicator of dispersion of values in a group. It is expressed in units which are the squared unit of these values.

The variance is valid for values with any distribution. See Example 6.42.2.

The variance of a group of values is equal to the squared deviation of this group.

The properties of the variance $\dot{V}$ are listed in the following text. Variables are denoted by x and y, constants by C and D, function (7.1) by $\dot{\mathrm{V}}(a)$, and the standard covariance of x and y, given by (7.21), by $\mathrm{V}(x, y)$.

- $\dot{V} \geq 0$. (7.6)
- $\dot{\mathrm{V}}(\mathrm{C}) = 0$. (7.7)
- $\dot{\mathrm{V}}(\mathrm{C} + x) = \dot{\mathrm{V}}(x)$. (7.8)
- $\dot{\mathrm{V}}(\mathrm{C} \cdot x) = \mathrm{C}^2 \cdot \dot{\mathrm{V}}(x)$. (7.9)
- If x and y are uncorrelated:
  $\dot{\mathrm{V}}(\mathrm{C} \cdot x + \mathrm{D} \cdot y) = \mathrm{C}^2 \cdot \dot{\mathrm{V}}(x) + \mathrm{D}^2 \cdot \dot{\mathrm{V}}(y)$. (7.10)
- If x and y are correlated:
  $\dot{\mathrm{V}}(\mathrm{C} \cdot x + \mathrm{D} \cdot y) = \mathrm{C}^2 \cdot \dot{\mathrm{V}}(x) + \mathrm{D}^2 \cdot \dot{\mathrm{V}}(y) + 2 \cdot \mathrm{C} \cdot \mathrm{D} \cdot \mathrm{V}(x, y)$. (7.11)
- If x and y are uncorrelated:
  $\dot{\mathrm{V}}(x \cdot y) \approx y^2 \cdot \dot{\mathrm{V}}(x) + x^2 \cdot \dot{\mathrm{V}}(y)$. (7.12)

[GUM] C.2.20, C.3.2, C.2.12 [Box 2005] PP 27 [Hristov, communication] [Welford] [Ivković] PP 45

8 Tests and functions fitting

8.1 Detection of outliers

Results of statistical analyses are valid only if the analyzed values are without outliers. Therefore, if there is a significant possibility that outliers will occur, their detection and rejection should be a mandatory part of the statistical analysis.

A good practice is to consider as outliers and reject the values which differ from an arithmetic mean of a sample, $\bar{x}$, more than the triple standard deviation of the sample, s.

According to this practice, detection of outliers should be carried out by firstly rejecting the values which are supposed to be outliers. It is followed by computing the mean $\bar{x}$ and the standard deviation s. Finally, it should be determined whether the supposed outliers are out of the interval $\bar{x} \pm 3 \cdot s$, and if they are, the supposition is true, so $\bar{x}$ and s can be assumed as valid. If among the supposed outliers they are certain values which are not out of the interval, the procedure should be repeated without rejection of these values. In the case of values with the normal distribution, such practice is justified by the fact that the probability that a valid value is out of $\bar{x} \pm 3 \cdot s$ is equal to 0.27 %. ● In the case of 30 values with the T-distribution, the probability is 0.55 %; ● for 10 values, it is 1.5 %; ● for 5 values, it is 4.0 %.

The text continues on the next page.

If the previous manner of detection of outliers is not sufficiently reliable, the following procedure is recommended for a sample from a population with the normal distribution. This test determines, with a requested probability, whether a value with the largest difference from the arithmetic mean is an outlier. The test is performed in the following steps.

1) Check if the sample is from a population with the normal distribution. Checking can be done according to the instructions in 8.6 Estimating the type and parameters of a distribution. If the sample is from the normal population, the test can be applied.

2) Let n be the number of values in the sample, including the tested extreme value. Let P be the requested probability of the test result. Determine a coverage factor $k(\nu, P)$ for the T-distribution with the parameters: a degrees of freedom, ν, and the probability P. This factor can be obtained from Table 6.3. The parameter ν is given by (8.1), and for $\nu > 100$, it can be assumed that $\nu = \infty$.

3) Determine the truth (that is, validity) of the inequality (8.2). The extreme value is denoted by $x_{extreme}$. An arithmetic mean of the sample without the extreme value is $\bar{x}'$, and the standard deviation of the mean is $s_{\bar{x}}'$. If the inequality is true, with the probability P or larger, $x_{extreme}$ is an outlier.

$$\nu = n - 2 \tag{8.1}$$

$$\left| x_{extreme} - \bar{x}' \right| > k(\nu, P) \cdot s_{\bar{x}}' \tag{8.2}$$

4) The extreme value is rejected if it is determined to be an outlier. The test is then repeated for each of the following extreme values, until finding a value which is not an outlier.

9 Measurement uncertainty

9.1 Measurement result and its uncertainty

Measurement uncertainty (**uncertainty**, or **uncertainty of measurement**) is a parameter of measurement result that describes the accuracy of the result by an indicator of dispersion of values that can be reasonably taken as the measurement result.

A measurement result can be used to compute the level and interval of confidence, if the result is the best estimated value of a measurand, and if that result is associated with the measurement uncertainty which is the standard deviation of that best estimated value, as well as the degrees of freedom of that deviation.

The best estimated value of a measurand, the measurement uncertainty of the evaluation, and the degrees of freedom of this uncertainty, can be used as component values in evaluating the uncertainty and degrees of freedom of another value in which the first value is used. This allows traceability.

In order to establish uniform estimating and expressing of measurement results, in 1993 ISO published the first edition of **Guide to the expression of uncertainty in measurement**, **GUM**. The Guide is applicable at various levels of accuracy, from a shop floor to the fundamental sciences. The basis of the Guide is the internationally adopted Recommendation of CIPM (from 1981) which accepts the Recommendation of BIPM Working Group on the Statement of Uncertainties (from 1980). Today, publishing and correction of the Guide are within the competence of the JCGM. The JCGM member organizations are: BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML.

The application of measurement uncertainty and traceability are mandatory for accredited laboratories, as well as organizations that have the quality management system according to ISO 9001:2008.

The summarized overview of the procedures given in the GUM for estimating of a measurement result and its measurement uncertainty is as follows.

● A measurement result is obtained based on the results of repeated measurements by determining the best estimated value of a measurand.

● For this best estimated value, the standard deviation, called the standard measurement uncertainty, is determined.

● For this standard uncertainty, the degrees of freedom is determined.

These evaluations can be obtained by using classical statistical methods (including convolution) which are regarded as the primary methods, or where appropriate, by using Monte Carlo simulation.

Standard measurement uncertainty (**standard uncertainty**) is the standard deviation of a measurement result which is the best estimated value of a measurand. Depending on the method for estimating the uncertainties, they are grouped into two categories: ● the standard uncertainties which are computed by statistical analysis of results of repeated measurements, type A; and ● the standard uncertainties which are estimated by statistical analysis of reliable information, type B.

The squared standard uncertainty, that is, the standard variance, is the unbiased evaluation of the squared deviation of a population of measurement results, that is, the variance of a population of these results. See 7.2 Experimental standard variance.

The standard deviation is the biased evaluation of the deviation of a population of measurement results. The bias of the uncertainty increases as its degrees of freedom decreases. In the case of the normal distribution, the relative error of the evaluation is in the interval from 0 % to +8.5 % for the degrees of freedom from ∞ to 3. See 6.20 Experimental standard deviation. When the standard uncertainty with a small degrees of freedom is taken as the evaluation of the deviation of the population of measurement results, the bias of this uncertainty should be corrected.

The standard uncertainty of the best estimated value of a measurand arises only from random effects and uncertainties of corrections.

Component standard uncertainty (or **standard uncertainty component**, or **component of standard uncertainty**) is a standard uncertainty of a component value.

[GUM] 2.2.3, 0, PP v, Foreword, Annex A, 3.2, 4.1.4, 0.7 2) 3) 4), 2.3.1, 4.2.3, 4.2.6, 7.2.1 d), 4.1.5, 4.1.6, 2.3.2, 2.3.3, 8, C.2.21 [VIM 2] 3.9 [VIM 3] 2.26 [IEC 17025] 5.4.6.1, 5.4.6.2, 5.10.4.1 [ILAC] 4.8 [ILAC, OIML] 1., 2. [NVLAP] 5.4.6, Annex B.1 [ISO 10012] 7.3 [ISO 9001] 7.6 [GUM-S1] Introduction, 1

9.3 Type B standard measurement uncertainty

Type B standard measurement uncertainty (**type B standard uncertainty**) is the standard deviation, of the best estimated value of a measurand, estimated by statistical analysis of reliable information. For this uncertainty, we should also give its degrees of freedom if it is deemed useful. Unless otherwise indicated, it is understood that the best estimated value of a measurand is the maximum likelihood value of the measurand and that this evaluation has the normal distribution.

Type B standard uncertainty is denoted by u_B, and its degrees of freedom by ν_B .

The type B standard uncertainty is estimated if there is not enough measurement data to compute the type A standard uncertainty.

The best estimated value of a measurand for which we give the type B standard uncertainty is often obtained by taking a single measurement result or by estimate on the basis of a small amount of information.

The type B standard uncertainty is estimated on the basis of an approximated probability density function obtained by analysis of the probability that an event will occur. The evaluations of distribution, deviation, and degrees of freedom of deviation, must be estimated by scientific judgment based on available information. Among other means, information can be obtained from the following sources:

● previous measurement results

● experience or general knowledge of the properties of materials and devices for measurement

● specifications provided by manufacturers of materials and devices for measurement

● calibration reports or other certificates

● handbooks with measurement uncertainties assigned to the given values.

Estimate of the type B standard uncertainty needs to be comprehensive so that the obtained evaluation is approximately as accurate as the type A standard uncertainty. This aim is easily achieved when the type A standard uncertainty is estimated for an evaluation obtained from a small number of averaged results. Section 6.22 Deviation of the standard deviation, Table 6.1, shows that the deviation of the standard deviation of an arithmetic mean is not negligible in practical cases.

The degrees of freedom of the standard uncertainty of type B can be computed according to: ● 6.24 Degrees of freedom; ● 6.25 Effective degrees of freedom; ● 6.28 Standard deviation and its degrees of freedom from the level and interval of confidence; or ● 6.26 Degrees of freedom from the deviation of the standard deviation of the arithmetic mean. If it is considered that the type B standard uncertainty is absolutely reliable, we assign to it the infinite degrees of freedom. See the following Examples 9.3.1 and 9.3.2.

[GUM] 0.7, 8, 3.1.2, 3.2.4, 4.3, 7.2.1 d), 7.2.7 c), G.4.2, 3.3.5, 4.3.2, E.4.3, G.4.2, G.6.4

9.3.1 Example

An ambient temperature is measured by a digital thermometer placed in an appropriate position. The thermometer is from a reliable manufacturer, with the resolution of 0.1 ºC. However, information about its accuracy is not available. The ambient temperature is achieved naturally (without using air conditioning).

For the two cases given below, estimate the environment temperature, its standard uncertainty and the degrees of freedom of the uncertainty.

a) The thermometer constantly gives 25.0 ºC.

b) The thermometer alternatively gives 25.0 ºC and 25.1 ºC, in approximately equal durations.

Solutions

a) The evaluation of the environment temperature t_a is given by:

t_a = *thermometer_indication* − *systematic_error* − *absolute_error_of_resolution*.

The furthermore evaluations are as follows. The systematic error has the normal distribution and its maximum likelihood arithmetic mean is equal to zero. The absolute error due to resolution has the uniform distribution and has the maximum likelihood value equal to zero. According to 6.12 Normal distribution, the resulting distribution of the estimated temperature is approximately normal. Therefore, we conclude that the maximum likelihood environment temperature is:

t_a = 25.0 − 0 − 0 = 25.0 ºC.

Low-accuracy thermometers normally have a maximum systematic error equal to their resolution. This maximum error probably matches the double standard deviation, so the estimated standard deviation of the systematic error is:

s_1 = 0.1 ºC / 2 = 0.050 ºC.

The standard deviation of the error due to resolution is computed from (6.95):

$$s_2 = \frac{0.1\,°C/2}{\sqrt{3}} = 0.029\ °C.$$

The standard uncertainty of the estimated environment temperature is computed from (6.41):

$$u_{Ba} = s_a = \sqrt{s_1^2 + s_2^2} = \sqrt{0.050^2 + 0.029^2} = 0.058\ °C.$$

The thermometer constantly indicates 25.0 ºC, and this is equivalent to an infinite number of results, *n*, from which the evaluations are obtained. The degrees of freedom of the uncertainty u_{Ba} is computed from (6.64):

$$\nu_{Ba} = n - 1 = \infty - 1 = \infty.$$

b) The estimated environment temperature t_b, is given by:

t_b = *thermometer_indication − systematic_error − absolute_error_of_evaluation_of_indication*.

The furthermore evaluations are as follows. The systematic error and the absolute error of evaluation of indication have the normal distributions, and the arithmetic means are equal to zero. According to 6.12 Normal distribution, the resulting distribution of the estimated temperature is approximately normal.

Considering the principle of operation of the analog-to-digital converter, the thermometer indication is the arithmetic mean of the two values which the thermometer alternatively gives:

$$thermometer\_indication = \frac{25.0\ °C + 25.1\,°C}{2} = 25.05\ °C.$$

So the maximum likelihood environment temperature is:

$$t_b = 25.05 - 0 - 0 = 25.05\ ºC.$$

(Like the case a.) Low-accuracy thermometers normally have a magnitude of the maximum systematic error equal to their resolution. This maximum error probably matches the double standard deviation, so the estimated standard deviation of the systematic error is:

$$s_3 = 0.1\ ºC / 2 = 0.050\ ºC.$$

The thermometer alternatively gives two values that are seemingly of equal durations, probably until the durations start to differ more than

20 % (0.2). Therefore, the absolute error of evaluation of indication has the approximate uniform distribution with the half-width equal to (0.2 / 2) · *resolution* = 0.01 ºC, so the standard deviation of this error is computed from (6.95):

$$s_4 = \frac{0.01\,°C}{\sqrt{3}} = 0.0058\ °C.$$

The standard uncertainty of the estimated environment temperature is computed from (6.41):

$$u_{Bb} = s_b = \sqrt{s_3^{\,2} + s_4^{\,2}} = \sqrt{0.050^2 + 0.0058^2} = 0.050\ °C.$$

The thermometer constantly indicates 25.05 ºC, and this is equivalent to an infinite number of results, *n*, from which the evaluations are obtained. The degrees of freedom of the uncertainty u_{Bb} is computed from (6.64):

$$\nu_{Bb} = n - 1 = \infty - 1 = \infty.$$

9.3.2 Example

It is estimated that out of 12 measurement results of length of one-type elements, half of the results (which are not individually known) have values in the interval from 10.07 mm to 10.15 mm. The results are from a population with the normal distribution (because errors of the results are a consequence of a large number of independent individual errors with a small share). The evaluation of the interval has a negligible systematic error.

Estimate: 1) the most frequent length of the elements, 2) the standard uncertainty of the evaluation of this length and 3) the degrees of freedom of this uncertainty.

Solution

1) The most frequent length is the mode. The estimated confidence limits are *a* = 10.07 mm and *b* = 10.15 mm. The results of measurement of the length have the T-distribution, so the value of the mode, *l*, is in the midpoint of the interval from *a* to *b*:

$$l = \frac{a+b}{2} = \frac{10.07\text{ mm} + 10.15\text{ mm}}{2} = 10.11\,\text{mm}.$$

3) The estimate of the standard uncertainty is carried out from total *n* = 12 results. Based on this, we compute the degrees of freedom of this uncertainty from (6.80):

$$\nu_B = n - 1 = 12 - 1 = 11.$$

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Chapters

From the Preface

From a shop floor to the fundamental sciences, statistical analysis of results of repeated measurements is required. The purpose of the analysis is to determine the value of a measured quantity, as well as the accuracy of the determined value. It is requested that accuracy is described by the standard measurement uncertainty to enable traceability and ensure computing of the confidence level and the confidence interval.

The essence of measurement is the same in all areas. The purpose of this publication is to be a companion to people dealing with measurements, that is experiments or observations, from production to research, from physics and chemistry to biology and medicine. The publication is also intended for students.

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