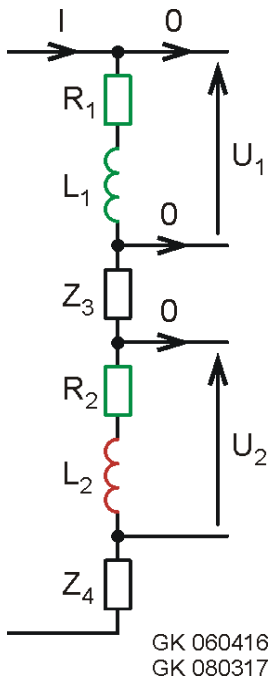


# Cetvorozlično merenje reaktanse na osnovu faznog stava napona

Symmetry, Goran Kostic, 080317



$$f := 50.253 \cdot 10^3 \quad \omega := 2 \cdot \pi \cdot f$$

$$\phi := -85.14 \quad \text{fazni stav } U_2 \text{ u odnosu na } U_1, [\text{deg}]$$

$$\phi := \frac{\pi}{180} \cdot \phi \quad \phi = -1.48597 \quad [\text{rad}]$$

$$\phi := -\phi \quad \text{fazni stav } U_1 \text{ u odnosu na } U_2$$

$$L_1 := 1.9372 \cdot 10^{-6} \quad \text{redna}$$

$$R_1 := 0.01142 \quad \text{redna}$$

$$R_2 := 17.965 \cdot 10^{-3} \quad \text{redna}$$

$$X_1 := \omega \cdot L_1$$

$$Z_1 := R_1 + iX_1 \quad \text{izvodjenje}$$

$$Z_2 := R_2 + iX_2$$

$$\arg \frac{U_1}{U_2} = \arg \frac{I \cdot Z_1}{I \cdot Z_2}$$

$$\arg U_1 - \arg U_2 = \arg Z_1 - \arg Z_2$$

$$\phi = \arg U_1 - \arg U_2$$

$$\phi = \arg Z_1 - \arg Z_2$$

$$\phi = \operatorname{atan} \left( \frac{X_1}{R_1} \right) - \operatorname{atan} \left( \frac{X_2}{R_2} \right)$$

$$X_2 := R_2 \cdot \tan \left( \operatorname{atan} \left( \frac{X_1}{R_1} \right) - \phi \right)$$

$$X_2 = 1.19 \times 10^{-3}$$

$$L_2 := \frac{X_2}{\omega}$$

$$L_2 = 3.76948 \times 10^{-9}$$

**vrednost redne induktivnosti  $L_2$**

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$$C_2 := -\frac{1}{\omega \cdot X_2}$$

$$C_2 = -2.661 \times 10^{-3}$$

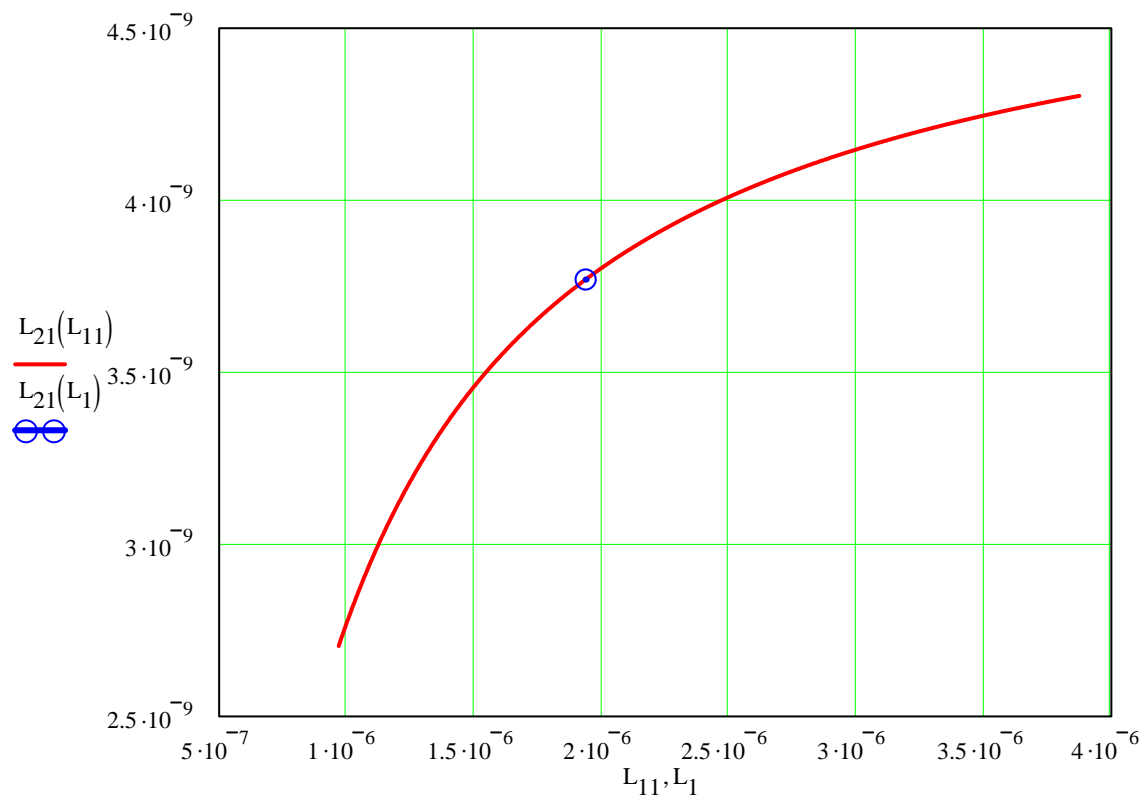
**vrednost redne kapacitivnosti  $C_2$**

## Greske pri odredjivanju induktivnosti $L_2$

$$L_2 := \frac{R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi\right)}{2 \cdot \pi \cdot f}$$

$$L_{11} := 0.5 \cdot L_1, 0.501 \cdot L_1 \dots 2 \cdot L_1$$

$$L_{21}(L_{11}) := \frac{R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_{11}}{R_1}\right) - \phi\right)}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dL1 := R_2 \cdot \frac{\left(1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)\right)^2}{R_1 \cdot \left(1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2}\right)}$$

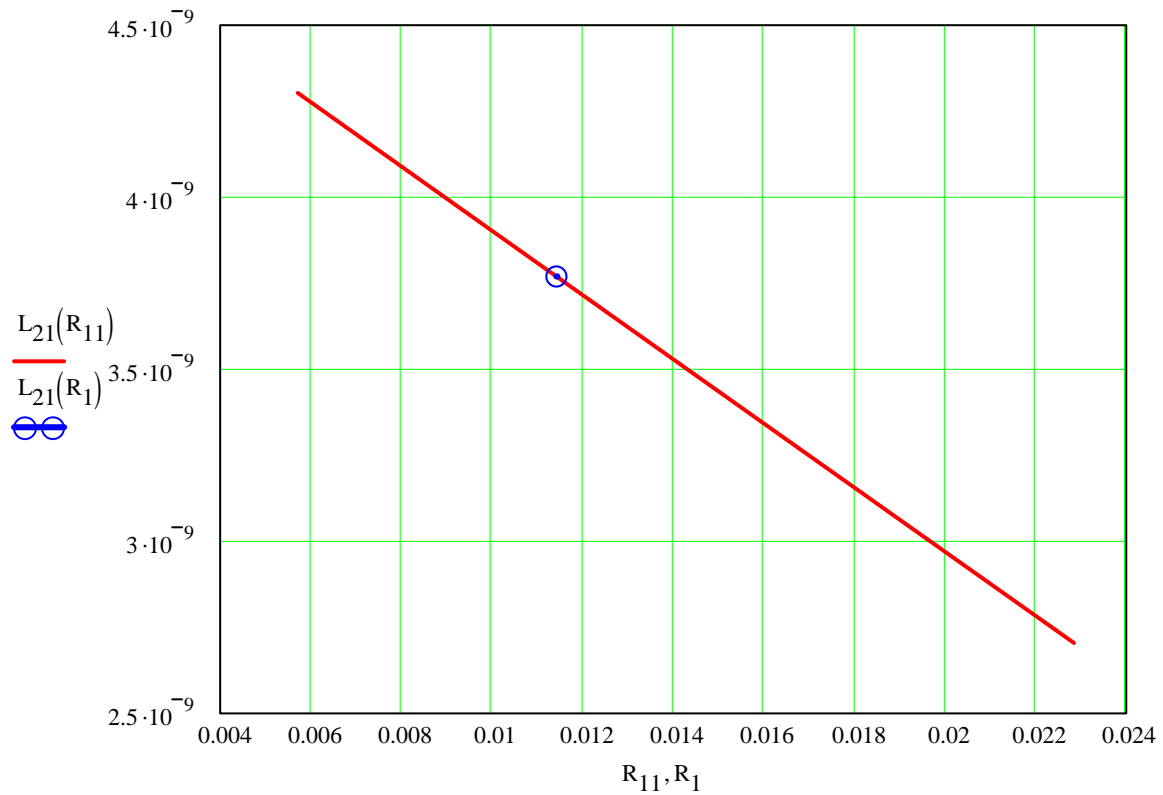
$$dL2\_po\_dL1 = 5.506 \times 10^{-4}$$

$$100 \cdot \frac{dL2\_po\_dL1}{L_2} \cdot \frac{1}{100} \cdot L_1 = 0.283$$

promena  $L_2$  u % za 1 % promene  $L_1$

$$R_{11} := 0.5 \cdot R_1, 0.501 \cdot R_1 \dots 2 \cdot R_1$$

$$L_{21}(R_{11}) := \frac{R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_{11}}\right) - \phi\right)}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dR1 := -R_2 \cdot \left(1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)\right)^2 \cdot \frac{L_1}{R_1^2 \cdot \left(1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2}\right)}$$

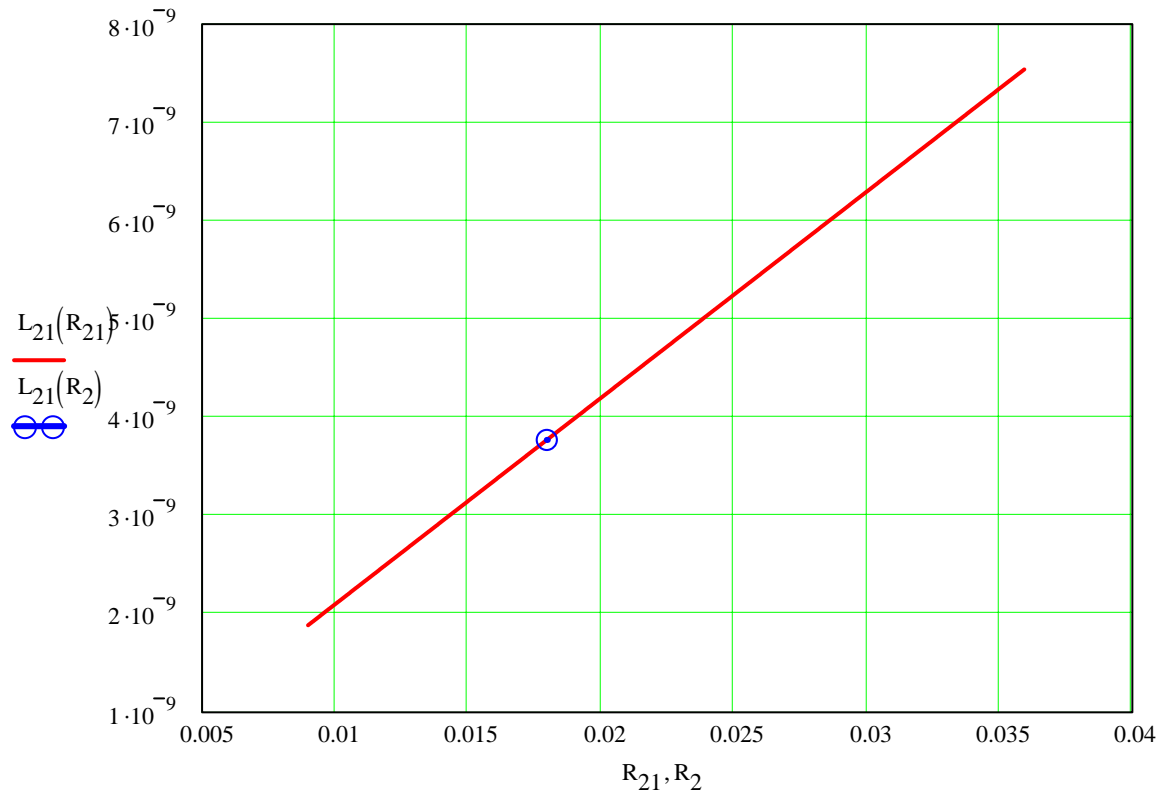
$$dL2\_po\_dR1 = -9.339 \times 10^{-8}$$

$$100 \cdot \frac{dL2\_po\_dR1}{L_2} \cdot \frac{1}{100} \cdot R_1 = -0.283$$

promena  $L_2$  u % za 1 % promene  $R_1$

$$R_{21} := 0.5 \cdot R_2, 0.501 \cdot R_2 \dots 2 \cdot R_2$$

$$L_{21}(R_{21}) := \frac{R_{21} \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi\right)}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_dR2 := \frac{\tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)}{2 \cdot \pi \cdot f}$$

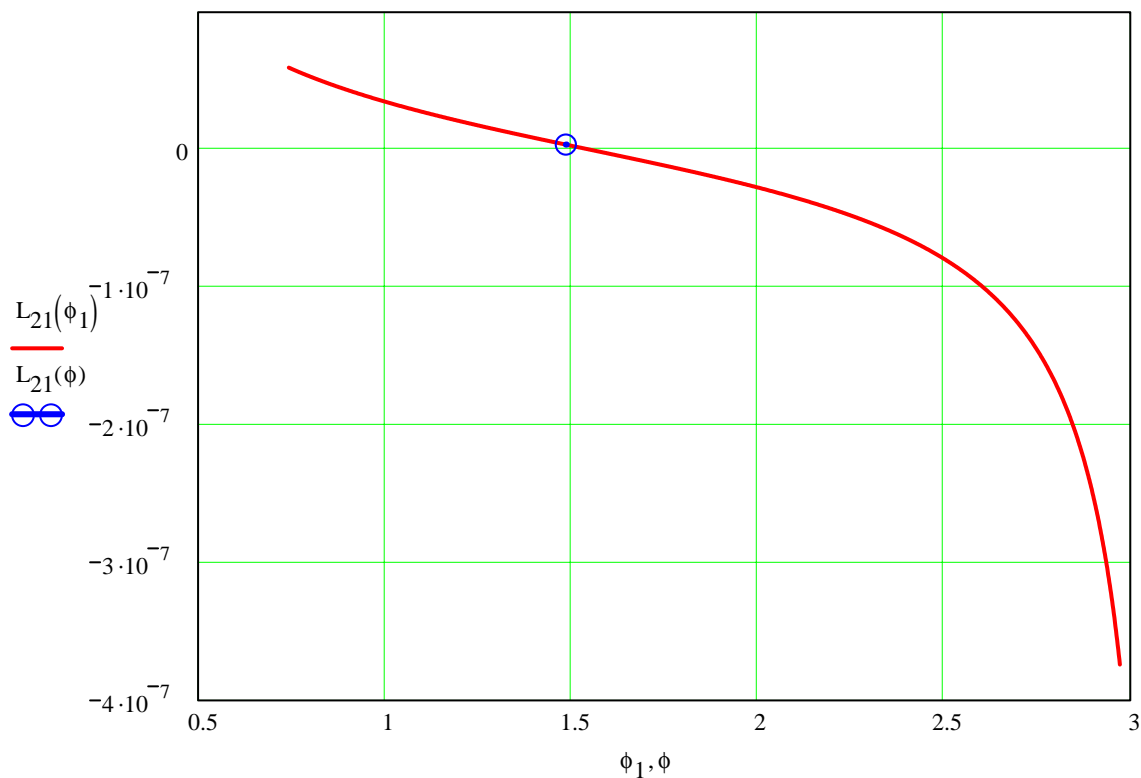
$$dL2\_po\_dR2 = 2.098 \times 10^{-7}$$

$$100 \cdot \frac{dL2\_po\_dR2}{L_2} \cdot \frac{1}{100} \cdot R_2 = 1$$

promena \$L\_2\$ u % za 1 % promene \$R\_2\$

$$\phi_1 := 0.5 \cdot \phi, 0.501 \cdot \phi \dots 2 \cdot \phi$$

$$L_{21}(\phi_1) := \frac{R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi_1\right)}{2 \cdot \pi \cdot f}$$



$$dL2\_po\_d\phi := R_2 \cdot \frac{-1 - \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2}{2 \cdot \pi \cdot f}$$

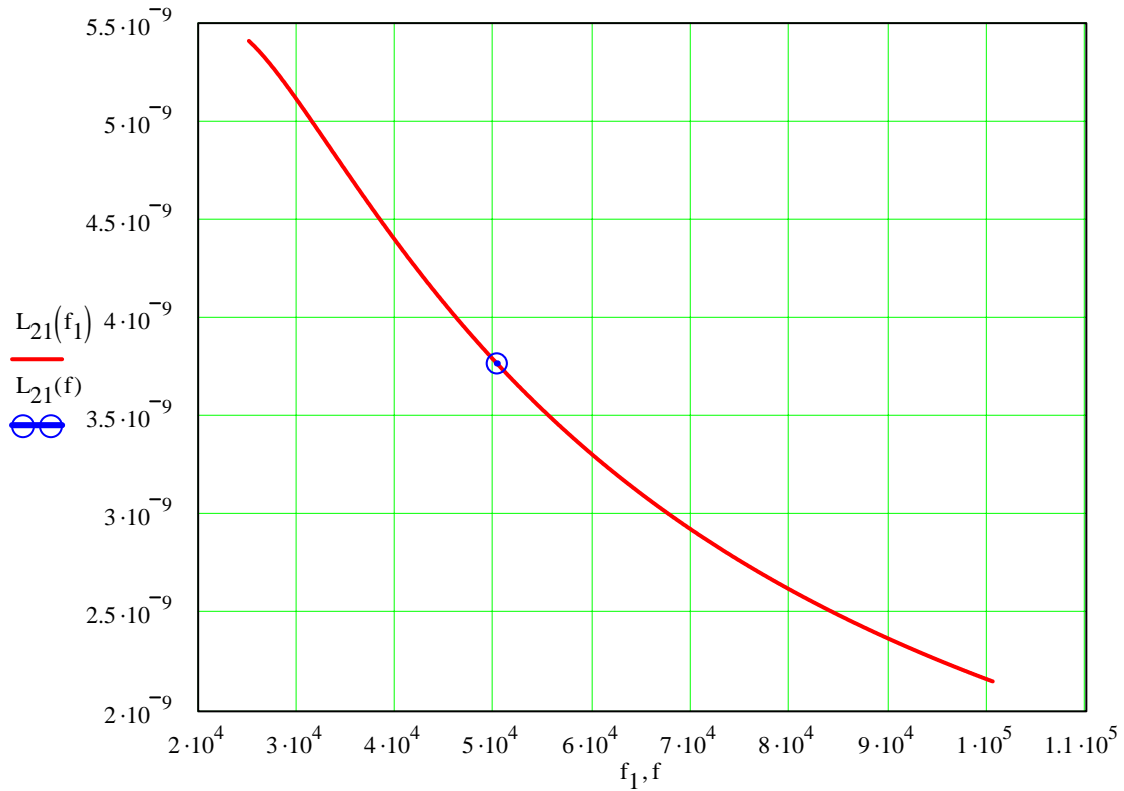
$$dL2\_po\_d\phi = -5.715 \times 10^{-8}$$

$$100 \cdot \frac{dL2\_po\_d\phi}{L_2} \cdot \frac{1}{100} \cdot \phi = -22.528$$

promena  $L_2$  u % za 1 % promene  $\phi$

$$f_1 := 0.5 \cdot f, 0.501 \cdot f \dots 2 \cdot f$$

$$L_{21}(f_1) := \frac{R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f_1 \cdot L_1}{R_1}\right) - \phi\right)}{2 \cdot \pi \cdot f_1}$$



$$dL_{2\_po\_df} := R_2 \cdot \left( 1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2 \right) \cdot \frac{L_1}{R_1 \cdot \left( 1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2} \right) \cdot f} - \frac{1}{2} \cdot R_2 \cdot \frac{\tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)}{\pi \cdot f^2}$$

$$dL_{2\_po\_df} = -5.379 \times 10^{-14}$$

$$100 \cdot \frac{dL_{2\_po\_df}}{L_2} \cdot \frac{1}{100} \cdot f = -0.717$$

promena \$L\_2\$ u % za 1 % promene \$f\$

## Kombinovana standardna nesigurnost pri odredjivanju induktivnosti $L_2$

$$u_{R\_L1} := 0.33 \text{ [%]}$$

$$u_{R\_R1} := 1.0 \text{ [%]}$$

$$u_{R\_R2} := 0.5 \text{ [%]}$$

$$u_{R\_φ} := 0.2 \text{ [%]}$$

$$u_{R\_f} := 0.0 \text{ [%]}$$

$$V_{L1} := \left( \frac{u_{R\_L1}}{100} \cdot L_1 \right)^2$$

$$V_{R1} := \left( \frac{u_{R\_R1}}{100} \cdot R_1 \right)^2$$

$$V_{R2} := \left( \frac{u_{R\_R2}}{100} \cdot R_2 \right)^2$$

$$V_{φ} := \left( \frac{u_{R\_φ}}{100} \cdot φ \right)^2$$

$$V_f := \left( \frac{u_{R\_f}}{100} \cdot f \right)^2$$

$$V_{C\_L2} := dL2\_po\_dL1^2 \cdot V_{L1} + dL2\_po\_dR1^2 \cdot V_{R1} + dL2\_po\_dR2^2 \cdot V_{R2} + dL2\_po\_dφ^2 \cdot V_{φ} + dL2\_po\_df^2 \cdot V_f$$

$$u_{C\_L2} := \sqrt{V_{C\_L2}}$$

$$u_{C\_L2} = 1.712 \times 10^{-10}$$

$$u_{RC\_L2} := 100 \cdot \frac{u_{C\_L2}}{L_2}$$

$$u_{RC\_L2} = 4.543 \quad \text{standardna relativna nesigurnost vrednosti } L_2 \text{ [%]}$$

$$3 \cdot u_{RC\_L2} = 13.629 \text{ [%]}$$

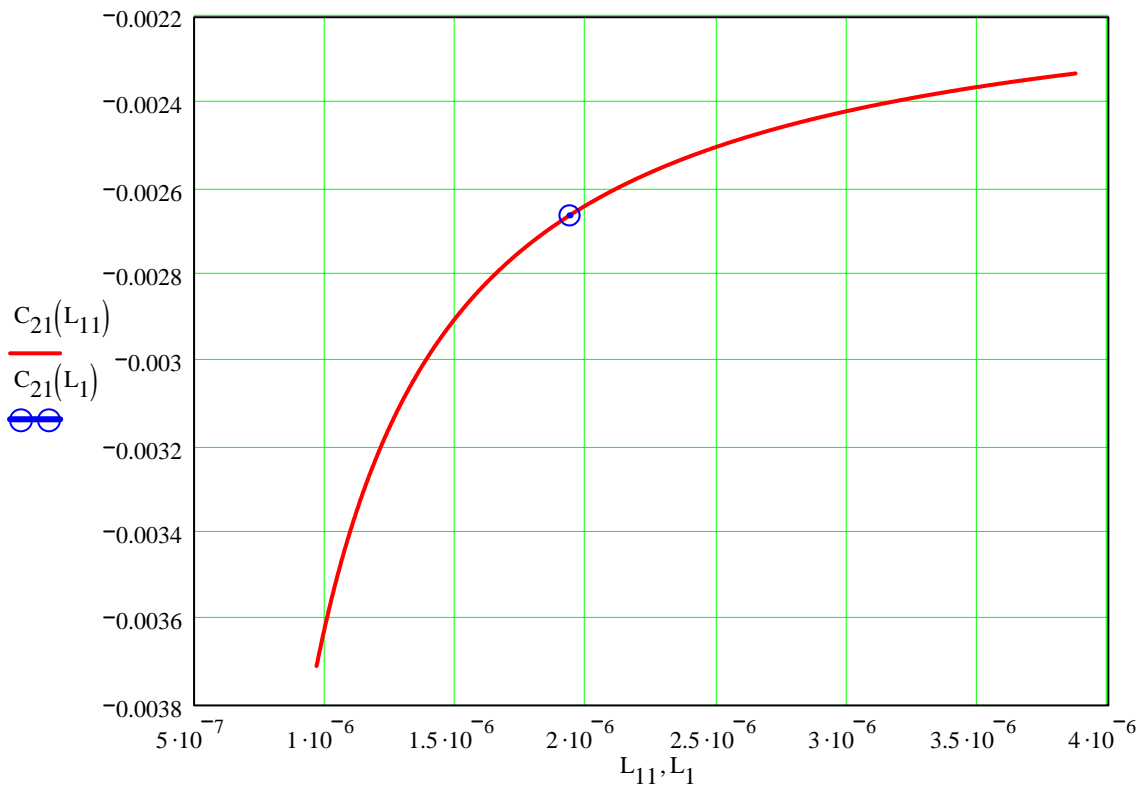
$$L_2 = 3.769 \times 10^{-9}$$

## Greske pri odredjivanju kapacitivnosti $C_2$

$$C_2 := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi\right)}$$

$$L_{11} := 0.5 \cdot L_1, 0.501 \cdot L_1 .. 2 \cdot L_1$$

$$C_{21}(L_{11}) := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_{11}}{R_1}\right) - \phi\right)}$$



$$dC2\_po\_dL1 := \frac{1}{R_2 \cdot \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2} \cdot \frac{\left(1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)\right)^2}{R_1 \cdot \left(1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2}\right)}$$

$$dC2\_po\_dL1 = 388.655$$

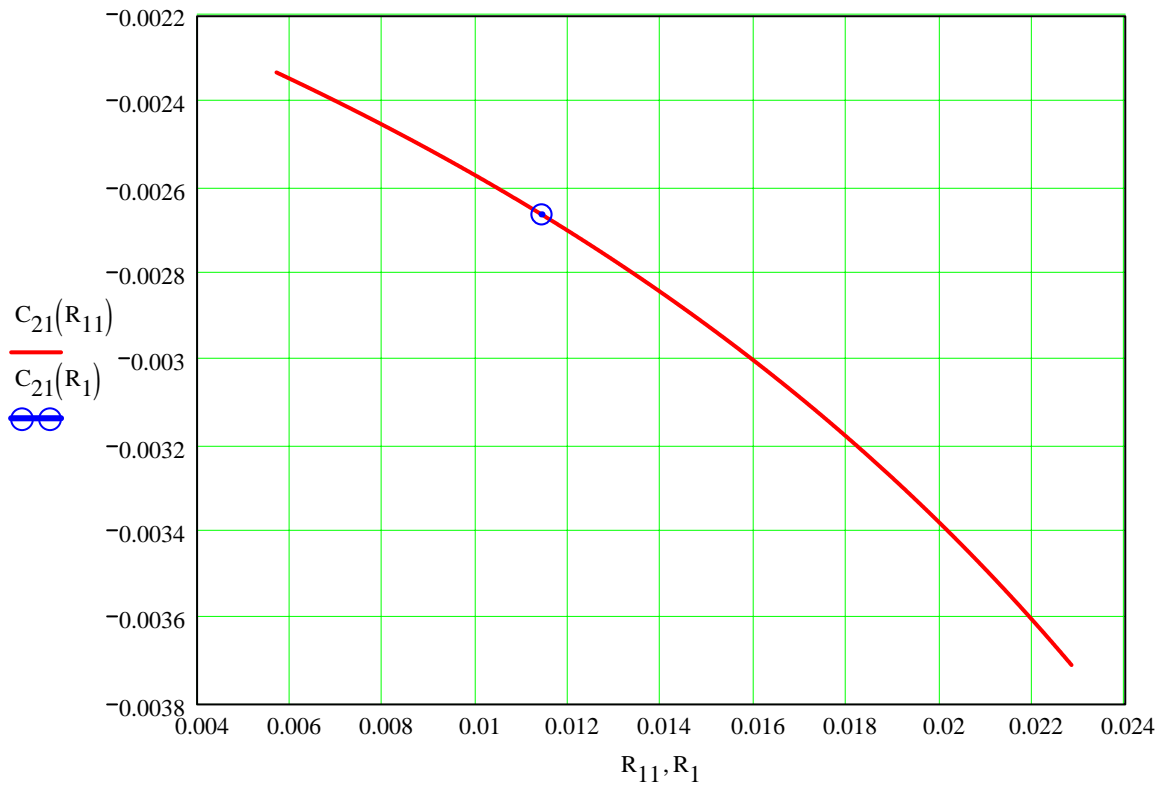
$$100 \cdot \frac{dC2\_po\_dL1}{C_2} \cdot \frac{1}{100} \cdot L_1 = -0.283$$

promena  $C_2$  u % za 1 % promene  $L_1$



$$R_{11} := 0.5 \cdot R_1, 0.501 \cdot R_1 \dots 2 \cdot R_1$$

$$C_{21}(R_{11}) := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_{11}}\right) - \phi\right)}$$



$$dC2\_po\_dR1 := \frac{-1}{R_2 \cdot \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2} \cdot \left(1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)\right)^2 \cdot \frac{L_1}{R_1^2 \cdot \left(1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2}\right)}$$

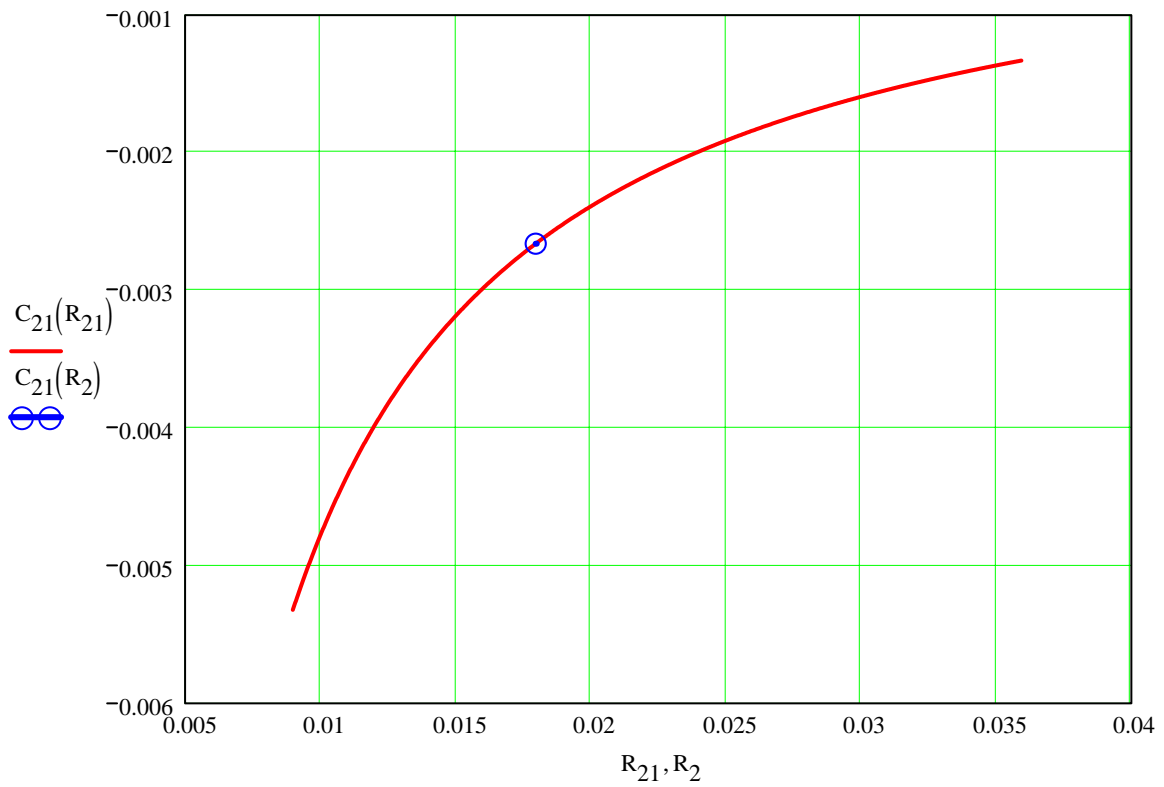
$$dC2\_po\_dR1 = -0.066$$

$$100 \cdot \frac{dC2\_po\_dR1}{C_2} \cdot \frac{1}{100} \cdot R_1 = 0.283$$

promena  $C_2$  u % za 1 % promene  $R_1$

$$R_{21} := 0.5 \cdot R_2, 0.501 \cdot R_2 \dots 2 \cdot R_2$$

$$C_{21}(R_{21}) := \frac{1}{2 \cdot \pi \cdot f \cdot R_{21} \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi\right)}$$



$$dC2\_po\_dR2 := \frac{1}{2 \cdot \pi \cdot f \cdot R_2^2 \cdot \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)}$$

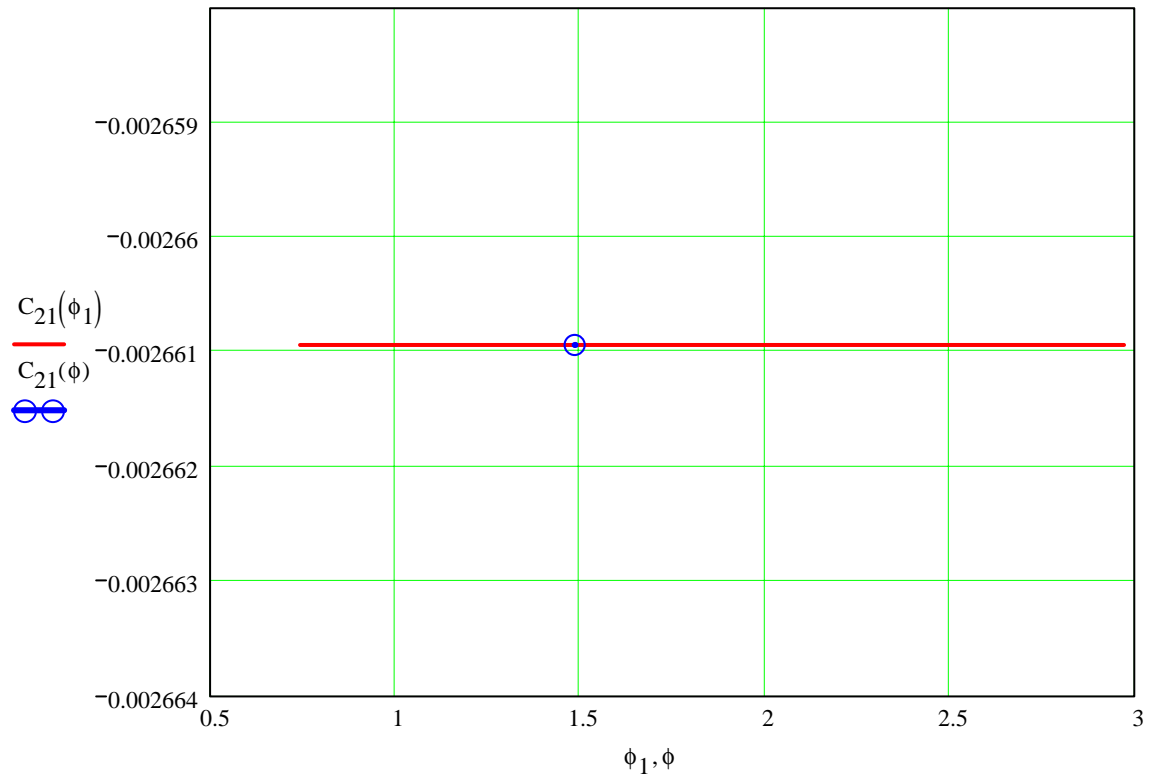
$$dC2\_po\_dR2 = 0.148$$

$$100 \cdot \frac{dC2\_po\_dR2}{C_2} \cdot \frac{1}{100} \cdot R_2 = -1$$

promena  $C_2$  u % za 1 % promene  $R_2$

$$\phi_1 := 0.5 \cdot \phi, 0.501 \cdot \phi .. 2 \cdot \phi$$

$$C_{21}(\phi_1) := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan \left( \operatorname{atan} \left( \frac{2 \cdot \pi \cdot f \cdot L_1}{R_1} \right) - \phi \right)}$$



$$dC2\_po\_d\phi := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan \left( \operatorname{atan} \left( 2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1} \right) - \phi \right)^2} \cdot \left( -1 - \tan \left( \operatorname{atan} \left( 2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1} \right) - \phi \right) \right)^2$$

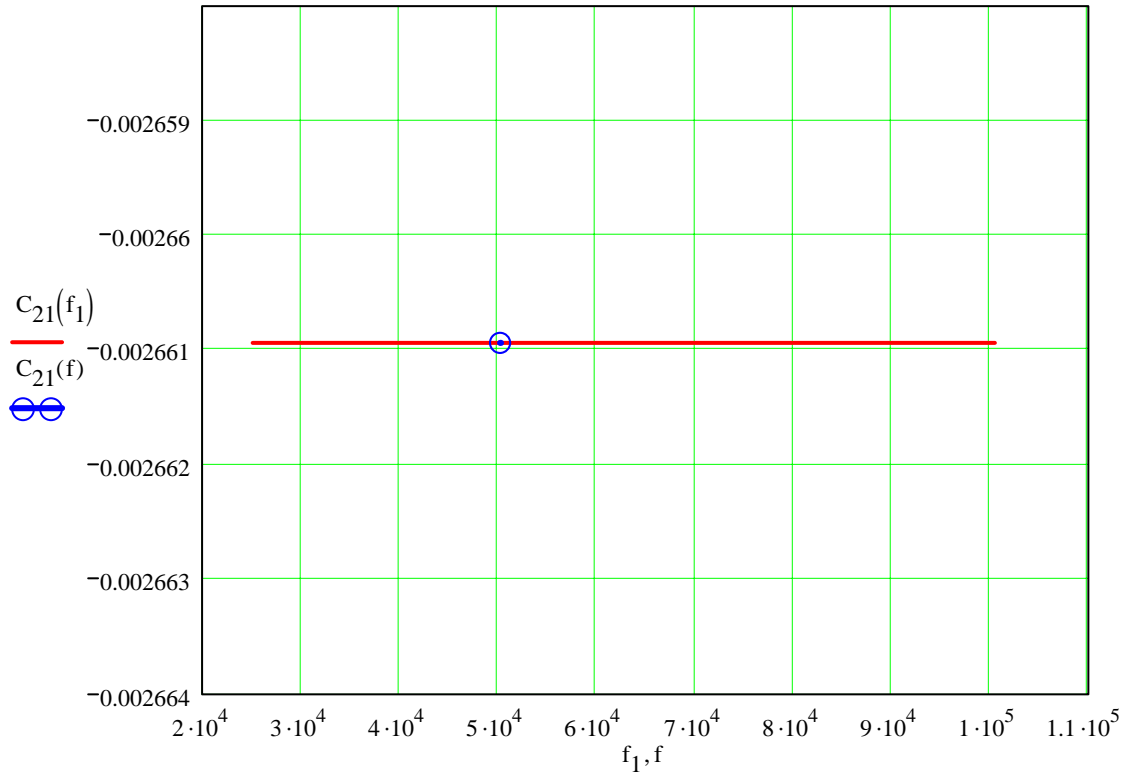
$$dC2\_po\_d\phi = -0.04$$

$$100 \cdot \frac{dC2\_po\_d\phi}{C_2} \cdot \frac{1}{100} \cdot \phi = 22.528$$

promena  $C_2$  u % za 1 % promene  $\phi$

$$f_1 := 0.5 \cdot f, 0.501 \cdot f .. 2 \cdot f$$

$$C_{21}(f_1) := \frac{1}{2 \cdot \pi \cdot f \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(\frac{2 \cdot \pi \cdot f \cdot L_1}{R_1}\right) - \phi\right)}$$



$$dC2\_po\_df := \frac{1}{2 \cdot \pi \cdot f^2 \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)} \dots$$

$$+ \frac{1}{f \cdot R_2 \cdot \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2} \cdot \left(1 + \tan\left(\operatorname{atan}\left(2 \cdot \pi \cdot f \cdot \frac{L_1}{R_1}\right) - \phi\right)^2\right) \cdot \frac{L_1}{R_1 \cdot \left(1 + 4 \cdot \pi^2 \cdot f^2 \cdot \frac{L_1^2}{R_1^2}\right)}$$

$$dC2\_po\_df = 6.793 \times 10^{-8}$$

$$100 \cdot \frac{dC2\_po\_df}{C_2} \cdot \frac{1}{100} \cdot f = -1.283$$

promena  $C_2$  u % za 1 % promene  $f$

## Kombinovana standardna nesigurnost pri odredjivanju kapacitivnosti $C_2$

$$u_{R\_L1} := 0.33 \text{ [%]}$$

$$u_{R\_R1} := 1.0 \text{ [%]}$$

$$u_{R\_R2} := 0.5 \text{ [%]}$$

$$u_{R\_φ} := 0.2 \text{ [%]}$$

$$u_{R\_f} := 0.0 \text{ [%]}$$

$$V_{L1} := \left( \frac{u_{R\_L1}}{100} \cdot L_1 \right)^2$$

$$V_{R1} := \left( \frac{u_{R\_R1}}{100} \cdot R_1 \right)^2$$

$$V_{R2} := \left( \frac{u_{R\_R2}}{100} \cdot R_2 \right)^2$$

$$V_{φ} := \left( \frac{u_{R\_φ}}{100} \cdot φ \right)^2$$

$$V_f := \left( \frac{u_{R\_f}}{100} \cdot f \right)^2$$

$$V_{C\_C2} := dC2\_po\_dL1^2 \cdot V_{L1} + dC2\_po\_dR1^2 \cdot V_{R1} + dC2\_po\_dR2^2 \cdot V_{R2} + dC2\_po\_dφ^2 \cdot V_{φ} + dC2\_po\_df^2 \cdot V_f$$

$$u_{C\_C2} := \sqrt{V_{C\_C2}}$$

$$u_{C\_C2} = 1.209 \times 10^{-4}$$

$$u_{RC\_C2} := 100 \cdot \frac{u_{C\_C2}}{C_2}$$

$$u_{RC\_C2} = -4.543 \quad \text{standardna relativna nesigurnost vrednosti } C_2 \text{ [%]}$$

$$3 \cdot u_{RC\_C2} = -13.629 \text{ [%]}$$

$$C_2 = -2.661 \times 10^{-3}$$